Black Holes and Quantum Info, Fall 2021
Problem Set 5

Due: in lecture Tuesday, Nov 16

Reading. Do the reading for each lecture posted on the website. Note that between lectures 18 and 19, I added a link to some notes on conformal transformations.

Problems

1. Do the exercise "Purification and the triangle inequality" in QGBH section 18.

2. Consider the conformal mapping

$$w = \left(\frac{w' - w_1}{w_2 - w'} \right)^{1/n} \tag{0.1}$$

where $n \geq 1$ is an integer. Calculate

$$\langle T'(w') \rangle \tag{0.2}$$

in the CFT on the w-plane with the metric $dwd\bar{w}$. Note that $\langle T(w) \rangle = 0$, so your answer will come entirely from the anomaly. You might want to look at the example on the last page of the 'To prime or not to prime' notes posted on the website under lecture 18. And for goodness sake please use Mathematica!

3. In two dimensions, the metric

$$ds^2 = b|z|^a dz d\bar{z} \tag{0.3}$$

can have a conical deficit or excess at the origin, depending on a, b. Calculate the circumference of a tiny circle of proper radius ϵ around the origin, and use your result to determine the defect angle as a function of a, b. The defect angle $\Delta\theta$ is defined by the relationship

$$\text{Circumference} = (2\pi + \Delta\theta)\epsilon \tag{0.4}$$

as $\epsilon \to 0$.

4. Consider n copies of the complex z plane, glued together cyclically along the real axes. The bottom of the real axis on copy k is glued to the top of the real axis on copy $k+1$ (with $n+1 \equiv 1$).

   If the metric is $dz d\bar{z}$ then this manifold has a conical excess $\Delta\theta = 2\pi(n-1)$ at the origin. But suppose the manifold is actually smooth; what is the metric near the origin?

   Hint: Consider $w = z^{1/n}$.

   (Comment: The Einstein equations imply that spacetime is locally flat — a conical excess is a curvature singularity. Therefore you can think of this problem as solving the Einstein equations at the point $z = 0$.)