5. Thermofield Double

Read: QGBH Thermofield double excercise in §5

QGBH 17.1

Entanglement

$|\psi\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2$ is <u>entangled</u> if

$|\psi\rangle \neq |\psi_1\rangle_1 \otimes |\psi_2\rangle_2$ for any $\psi_{1,2}$

ρ on $\mathcal{H}_1 \otimes \mathcal{H}_2$ is entangled if

$$\rho \neq \sum_i \omega_i \, \rho_1^{(i)} \otimes \rho_2^{(i)}$$

We'll have a <u>lot</u> more to say about entanglement, but that's all for now.

Thermofield Double

Any mixed state ρ in QM can be "purified"

by enlarging $\mathcal{H} \subset \mathcal{H} \otimes \mathcal{H}_{aux}$

i.e., $\exists$ $|\psi\rangle \in \mathcal{H} \otimes \mathcal{H}_{aux}$,

$$\rho_\psi = |\psi\rangle\langle\psi| \qquad \text{(pure!)}$$

$$\rho = \mathrm{tr}_{aux}\, \rho_\psi$$

Thus $\langle O_1 O_2 \cdots \rangle_\rho = \langle\psi| O_1 O_2 \cdots |\psi\rangle$

And we can take

$$\mathcal{H}_{aux} \cong \mathcal{H}$$

Proof:

Diagonalize $\rho = \sum_i p_i |i\rangle\langle i|$

Then

$$|\psi\rangle = \sum_i \sqrt{p_i}\, |i\rangle |i\rangle$$

exercise: $\mathrm{tr}_{aux} |\psi\rangle\langle\psi| = \rho$

* purification is not unique.
* entangled pure state in $\mathcal{H} \times \mathcal{H}_{aux} \xrightarrow{\mathrm{tr}_{aux}}$ mixed state in $\mathcal{H}$

Applied to thermal state $\rho = e^{-\beta H}$:

Conventionally for QFTs we take the CRT conjugate here, then $\mathcal{H}_1 = \mathcal{H}_2$

Double $\mathcal{H} \to \mathcal{H}_1 \otimes \mathcal{H}_2$ (identical copies of $\mathcal{H}$)

CRT

$$|\beta\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |n\rangle_1 |\tilde{n}\rangle_2$$

For $\mathcal{O}$'s acting only on $\mathcal{H}_1$,

$$\langle\beta| \mathcal{O}_1 \mathcal{O}_2 \cdots |\beta\rangle = \frac{1}{Z} \operatorname{Tr} e^{-\beta H} \mathcal{O}_1 \mathcal{O}_2 \cdots$$

This all works for any thermal system in QM.

Now return to QFT:

Note isomorphism

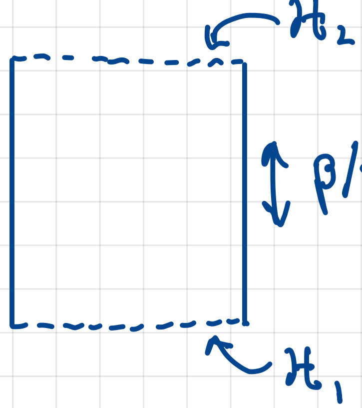
operators $\rho : \mathcal{H} \to \mathcal{H}$

states $\psi \in \mathcal{H} \otimes \mathcal{H}$

$$|m\rangle_1 |n\rangle_2 \longleftrightarrow |m\rangle\langle n|$$

$$\underset{\mathcal{H}\otimes\mathcal{H}}{|\beta\rangle} \longleftrightarrow \underset{\mathcal{H}\to\mathcal{H}}{\sqrt{\rho_\beta}}$$

TFD from Path integral

$|\beta\rangle =$ 

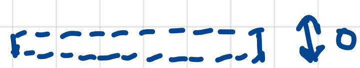
$= e^{-\beta H/2} = \sqrt{\rho_\beta}$

exercise: check $\langle \phi_1, \phi_2 | \beta \rangle$ is correct.

(ie agrees with definition of $|\beta\rangle$ above)

formally, "maximally entangled" state

$|max\rangle = \sum |n\rangle |n\rangle \longleftrightarrow$ operator $\mathbb{1}_{\mathcal{H}\to\mathcal{H}}$

$=$ 

and

$$|\beta\rangle = e^{-\beta H_1/4} e^{-\beta H_2/4} |max\rangle$$

So I like to think of TFD path integral this way.

Minkowski Vacuum = Rindler TFD

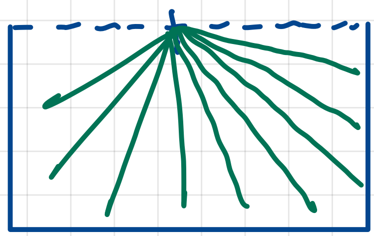
Claim:

$$|0\rangle_{\text{Mink}} = |\beta\rangle_{\text{Rind}\otimes\text{Rind}}$$

w.r.t. Boost charge.

Derivation:

$|0\rangle_{M.} =$



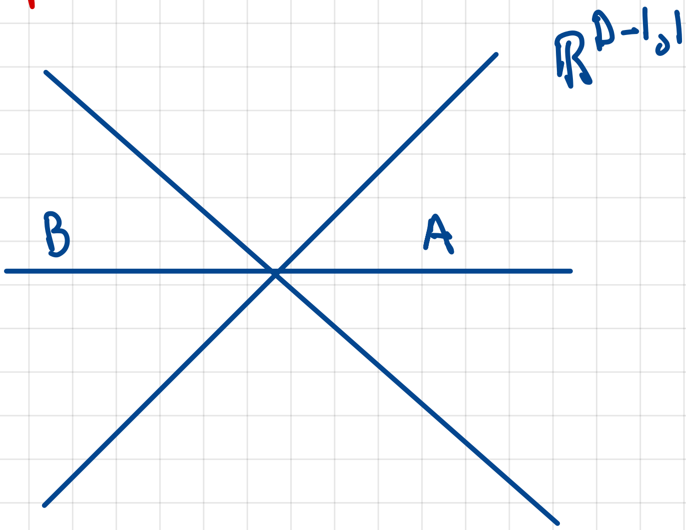
"strip of size $\beta/2$ in θ-direction"

Re-slice $\rightsquigarrow$ Path-integral rep. of $\sqrt{\rho_\beta} = e^{-\pi K_A}$

or

$$|\beta\rangle = \sum_n e^{-\pi K_A} |n\rangle |\tilde{n}\rangle$$

Interpretation



In QFT vacuum, B and A are highly entangled.

(∞ entanglement at short distances!)

This is why ρ_A alone looks thermal.

We'll revisit this more quantitatively later.

and why

$$\langle \phi(t=0, \vec{x})\, \phi(t=0, \vec{y}) \rangle \neq 0$$