15.

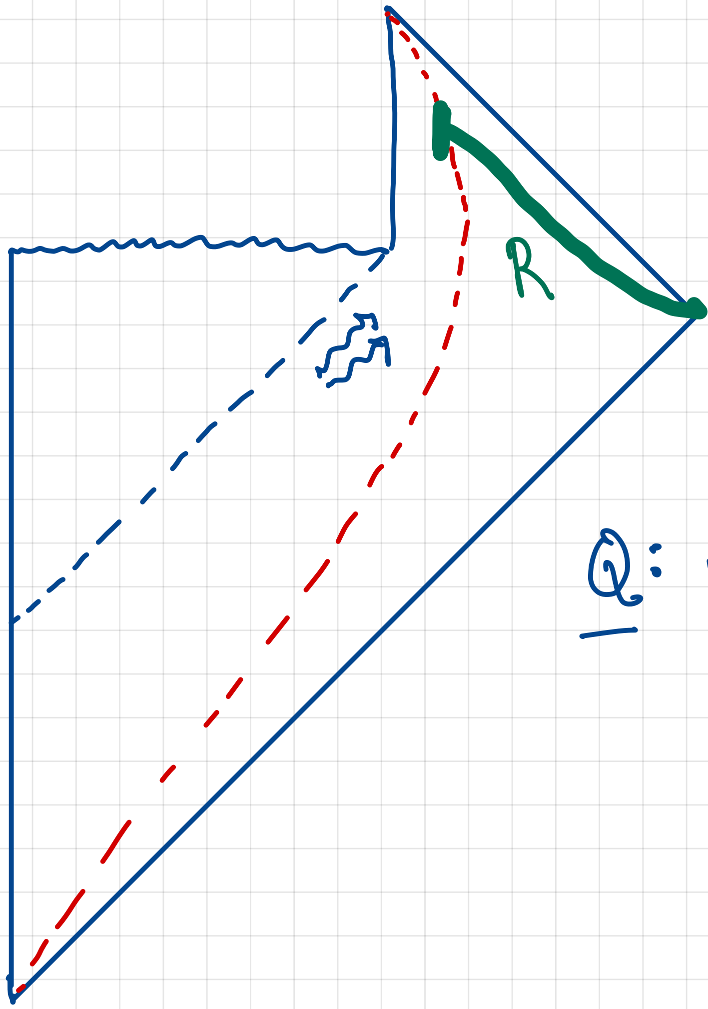
Plan:
- Page curve basics
- Page's argument
- Mathur theorem
- Firewalls.
- Sekino-Susskind ?
- Wall ?

QI in Hawking Radiation

Take 1

Read: Harlow Jerusalem Lectures, §5

Evaporating Black Hole

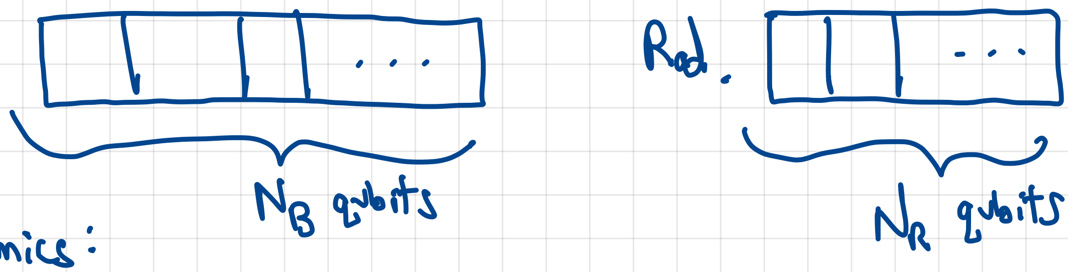


$R = \{ r > r_0 \, , \text{ time } t \}$

Q: what is $S(\rho_R(t))$?

(Rough) Qubit Model

BH. Rad.



Dynamics:

@ each time step,

1) $\rho_{BH} \to U^{\dagger} \rho_{BH} U$

random unitary

2) "Emit" one qubit w/ probability Γ

ie move qubit # N_B from BH $\to$ Rad.

Find $S(\mathrm{Rad}; t)$.

Claim:

Random states on $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ are nearly maximally mixed for $R \ll B$.

Defn. $\|M\|_1 = \operatorname{tr} \sqrt{M^\dagger M}$ trace norm

$\|M\|_2 = \sqrt{\operatorname{tr} M^\dagger M}$ L_2 norm

"distance" $\|\rho - \sigma\|$

$$\|M\|_2 \leq \|M\|_1 \leq \sqrt{N} \, \|M\|_2$$

"Random" State:

$$|\psi(U)\rangle = U|\psi_0\rangle$$

$\uparrow$ Haar random unitary

Page Theorem:

$$\int dU \left\| \rho_A(U) - \frac{1}{|A|} \mathbb{1}_A \right\|_1 \leq \sqrt{\frac{|A|^2 - 1}{|A||B| + 1}}$$

$$\leq \sqrt{\frac{|A|}{|B|}}$$

for $|A|, |B| \gg 1$

Proof: see Harlow.

"Haar" means

$$\int dU = 1$$

$$\int dU \, U_{ij} U^\dagger_{kl} = \frac{1}{N} \delta_{il} \delta_{jk} \qquad [\text{deriv}: \ \Sigma\ \delta\text{'s}, \ U^\dagger U = \mathbb{1}]$$

etc.

In terms of entropy:

$$\int dU \, S(\rho_A(U)) = \log|A| - \frac{1}{2}\frac{|A|}{|B|} + \text{subleading}$$

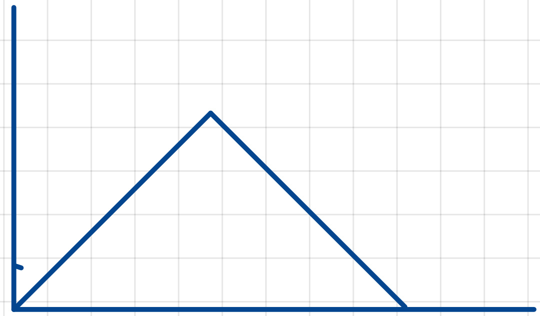
ex: $A = 1000$ qubits, $B = 1003$ qubits,

typical $S_A \sim 1000 \log 2 - \frac{1}{2} 2^{-3} + \ldots$

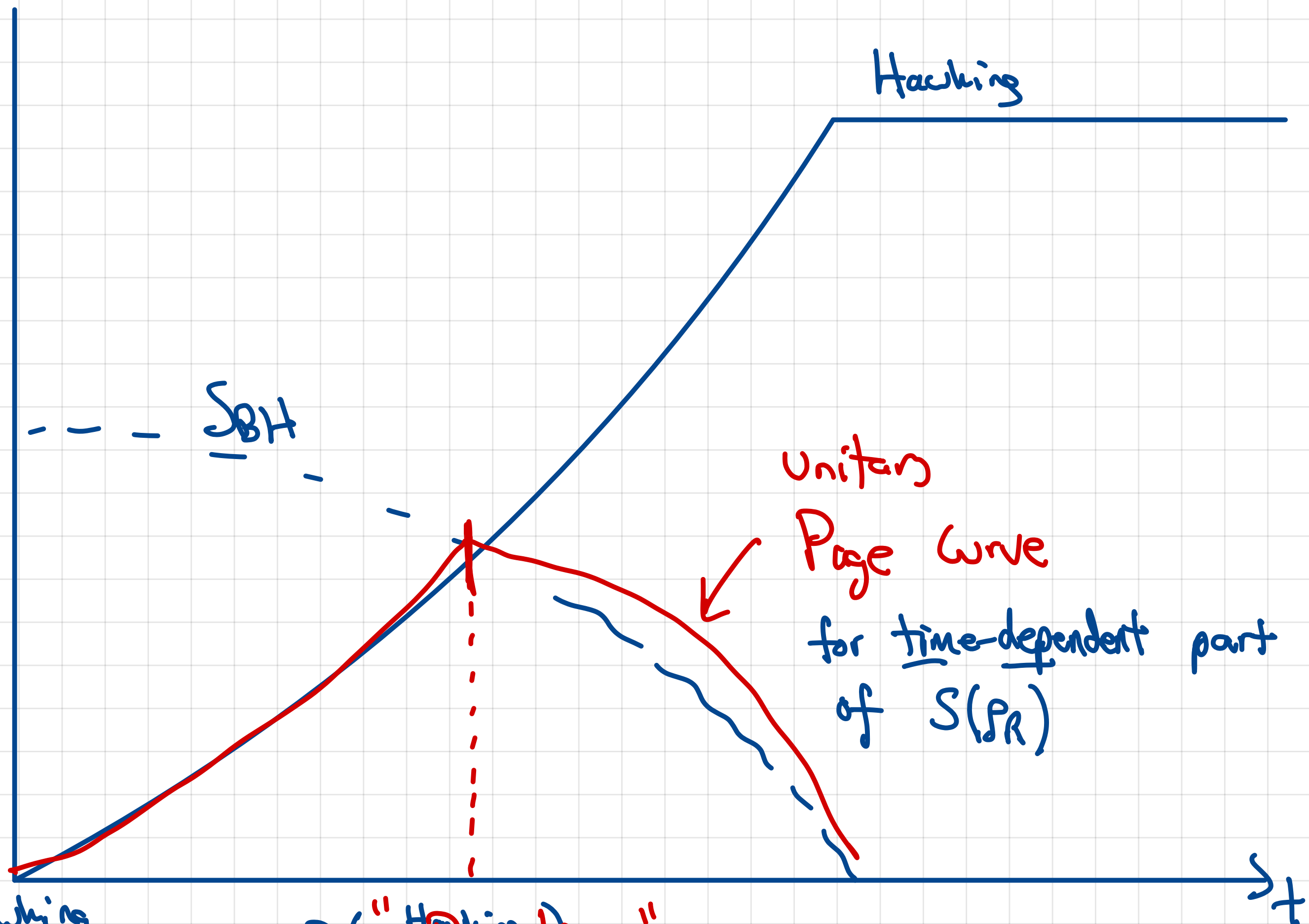
Qubit Page curve:

$$S(\text{Rad};t) \simeq \min\left\{ \log|BH|,\ \log|\text{Rad.}| \right\}$$

$$=$$



Page Curve for Hawking Radiation



Hawking curve $= S(\rho_A^{\text{Hawking}})$

"Page time"

$\approx$ thermodynamic entropy of radiation

$$\sum_{i=1}^{N_{\text{emitted}}} S_i(\beta(t_i))$$

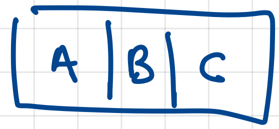
$$S_{BH} = \frac{\text{Area}}{4}$$

Mathur's Theorem

Small, local corrections to Hawking's calculation cannot fix the entropy.

Proof

Recall SSA, $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$

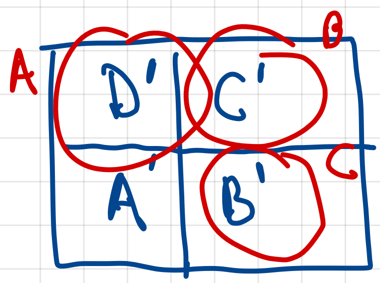


$$S_{AB} + S_{BC} \geq S_B + S_{ABC}$$

Equivalently,

$$S_{AB} + S_{BC} \geq S_A + S_C$$

Proof: Purify $A'B'C'$

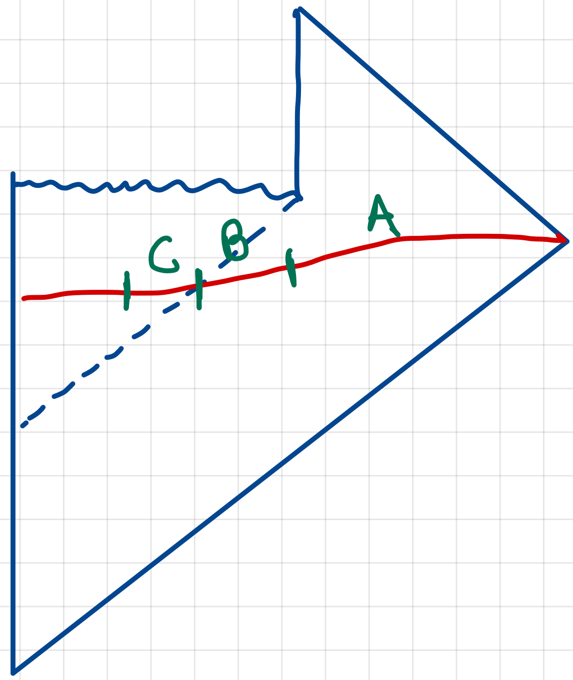


$$S_{A'B'} + S_{B'C'} \geq S_{B'} + S_{A'B'C'}$$

$$\Downarrow$$

$$S_{C'D'} + S_{C'B'} \geq S_{B'} + S_{D'}$$

✓



C
B
A

Escape of Hawking radiation $\Rightarrow$

$$S_A(t+\delta t) = S_{AB}$$

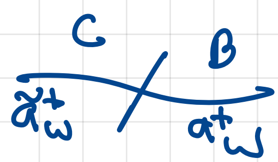
$$< S_A(t) \quad \text{for unitary evap. after page time}$$

Smoothness of horizon $\Rightarrow$

$$S_{BC} < S_B \approx S_C$$

$$\text{b/c} \quad |BC\rangle \sim \exp\left(-\int d\omega\, e^{-\beta\omega/2}\, a^\dagger_\omega \tilde{a}^\dagger_\omega\right)|0\rangle$$

(TFD)



<u>So</u>

$$S_{AB} < S_A$$

$$S_{BC} < S_B \approx S_C$$

This contradicts SSA!

$$S_{AB} + S_{BC} \geq S_A + S_C$$

"Monogamy" of entanglement: (cf. no cloning)

If a late-time Hawking particle is highly entangled with its interior partner, it cannot also be highly entangled w/ prior radiation

$\Rightarrow$ Page curve can only go up!

This is robust to any small corrections to quantum state $|BC\rangle$

"Firewall" (AMPS paradox v.1)

Page curve decreases

$\Rightarrow$ A, B entangled

$\Rightarrow$ B, C not (very) entangled

$\Rightarrow$ State @ horizon doesn't look like vacuum @ short distance

"Paradox" = Tension among

- Unitary QM
- EFT
- Locality

But what is locality in QG?

Loophole

A, B, C are NOT independent in quantum gravity

$$\mathcal{H} \neq \mathcal{H}_{\text{int}} \otimes \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

Why?

Non-perturbative effects:

$$\rho_R = \rho_R^{\text{Hawking}} \underbrace{(1 + \text{small})}_{\text{perturbative}} + \underbrace{e^{-\# S}}_{?}$$

perturbative $\Rightarrow$ respects $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$ factorization, cannot help paradox

This might be big enough!

$$S = -\operatorname{tr} \rho \log \rho$$

$$\simeq -\sum_{i=1}^{e^S} \lambda_i \log \lambda_i$$

AdS/CFT

Clearly this is how it works here if AdS/CFT is correct.

But you cannot say AdS/CFT is "demonstrating unitary evaporation"

Maybe Hawking's calculation disproves AdS/CFT!