11.

AdS/CFT

Read: QGBH §10, §14

Holographic Principle

't Hooft ; Susskind

"All d.o.f. in quantum gravity are associated to the boundary of spacetime"

① Observables @ ∞

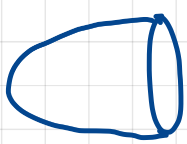
② $S_{BH} = \text{Area}/4$

In fact, Bekenstein:

$$S_{\text{anything}} \leq \frac{\text{Area}}{4}$$

(static, spherical sym., ...)

Path integral argument:

$Z(\beta) =$  G_β

This was a postulate about gravity. If gravity is equivalent to "QM on the boundary" then it becomes true by the ordinary $\text{Tr}\, e^{-\beta H}$ path integral argument.

(closed universes ???)

AdS/CFT duality

In asymptotically anti-de Sitter, this is true, and those "d.o.f." $\equiv$ Conformal field theory.

I will state this more carefully soon; 1st describe the players.

(Here d.o.f = local QFT. This is NOT true in general.

AdS_{d+1}

Max. symmetric, constant negative curvature

Embedded in $\mathbb{R}^{d,2}$:

$$-X_0^2 + X_1^2 + \cdots + X_d^2 - X_{d+1}^2 = -\ell^2$$

(Don't try to picture this ... 2 times!)

Solve by:

$$X_0 = \ell \cosh\rho \cos\tau$$

$$X_{d+1} = \ell \cosh\rho \sin\tau$$

$$X_i = \ell \sinh\rho \; n_i \qquad \left(\Sigma n_i^2 = 1 \Rightarrow S^{d-1} \right)$$

$\Longrightarrow$

$$ds^2 = \ell^2 \left[-\cosh^2\rho \, d\tau^2 + d\rho^2 + \sinh^2\rho \, d\Omega_{d-1}^2 \right]$$

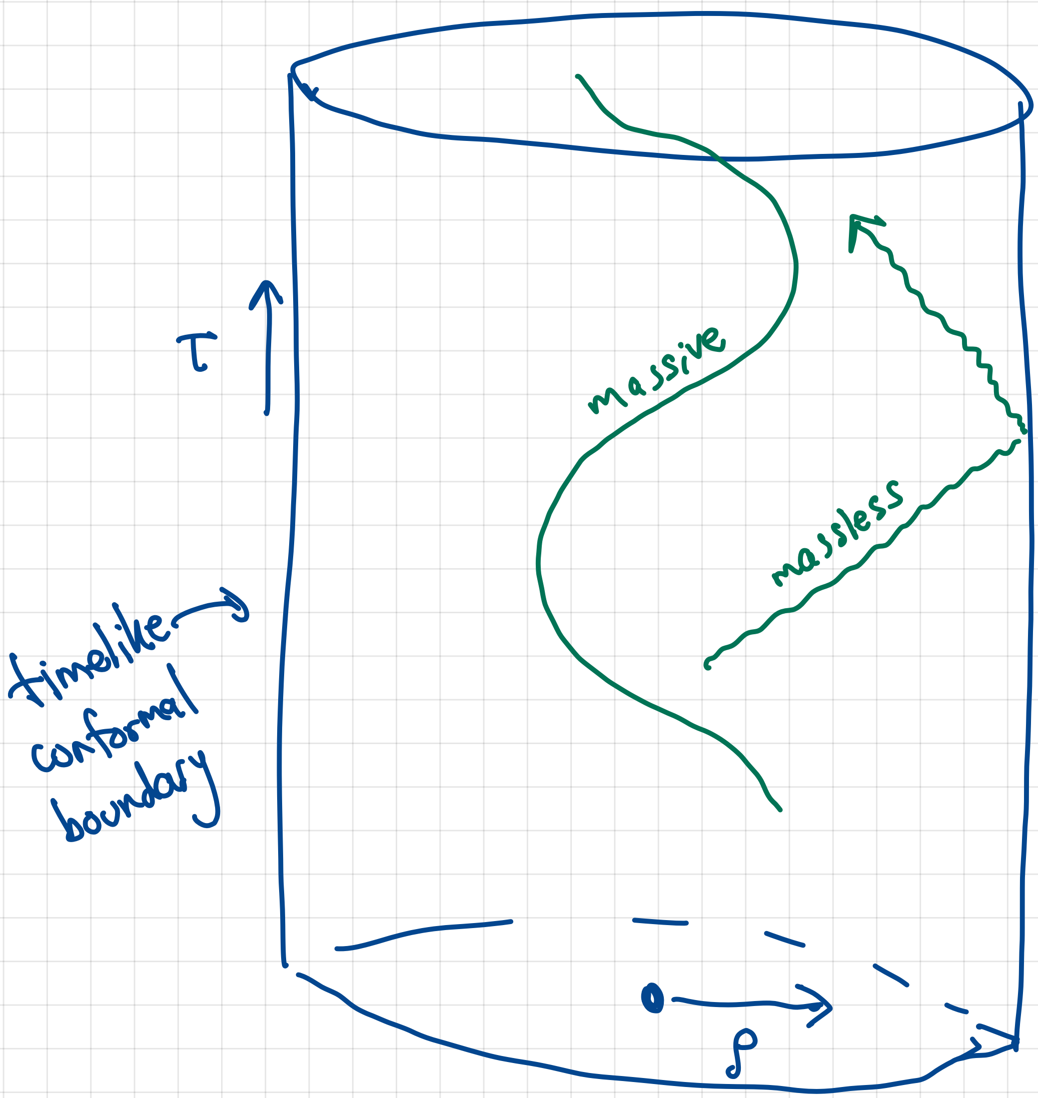
"global coords."

unwrap $\tau \in (-\infty, \infty)$

* Solves Einstein eqs with $\Lambda \propto -\frac{1}{\ell^2}$

* max. symmetric neg. curv.

Penrose diagram:



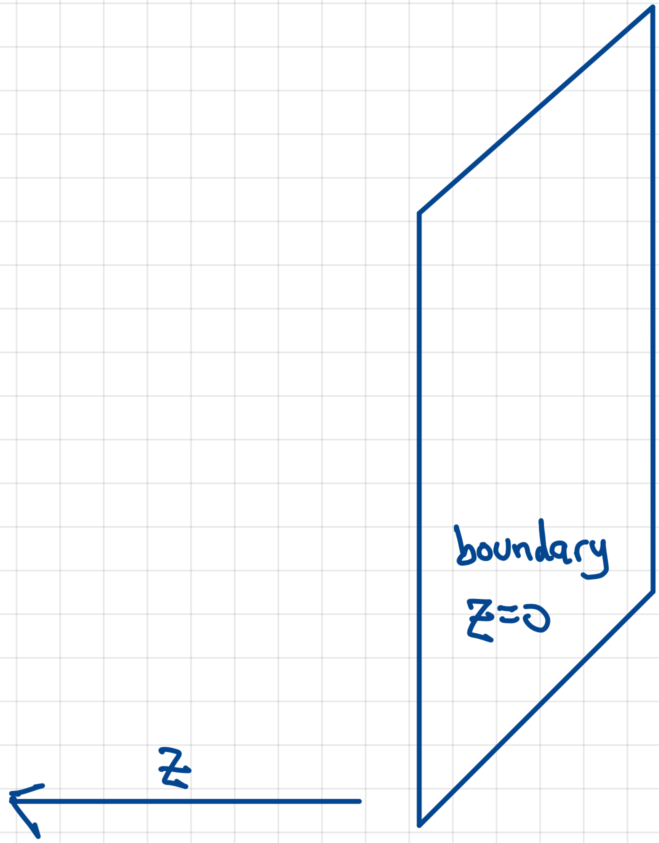
"Box"   massive particles see a potential $e^{2\rho}$

Note: "Lots of space near the boundary"

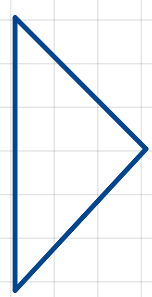
Poincare Patch

(Different way to solve embedding eqn.)

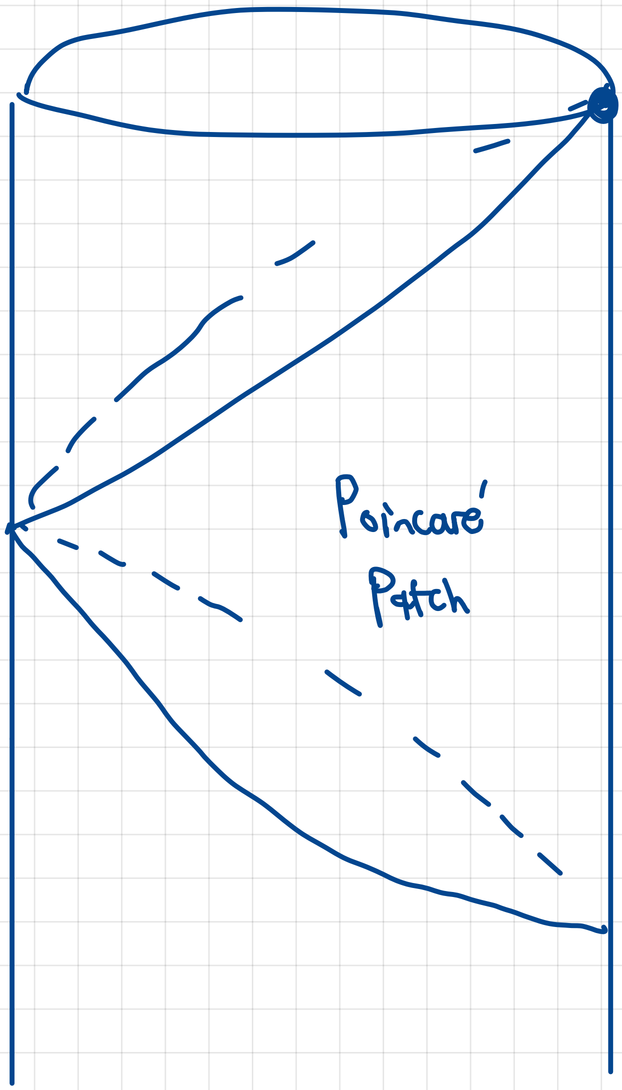
$$ds^2 = \frac{\ell^2}{z^2}\left(dz^2 - dt^2 + d\vec{x}_{d-1}^{\,2} \right)$$



The z=0 surface = Minkowski ⇒ Penrose diamond



Relation



Isometries: $SO(d,2)$

(see embedding! $X_0 \partial_{X_1} + X_1 \partial_{X_0}$, etc.)

CFT

$\equiv$ QFT with conformal symmetry

Ex: free massless scalar $\mathcal{L} = (\partial\phi)^2$ [always massless!]

Conformal group $\equiv$ diffeos such that

$$ds^2_{\text{mink.}} \to \Omega^2(x)\, ds^2_{\text{mink}}$$

under $x^\mu \to x^{\mu'}(x)$,

$$ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu$$

$$\to \eta_{\mu'\nu'}\, dx^{\mu'} dx^{\nu'} = \eta_{\mu'\nu'} \frac{\partial x^{\mu'}}{\partial x^\mu} dx^\mu \frac{\partial x^{\nu'}}{\partial x^\nu} dx^\nu$$

so
$$\left(\frac{\partial x'}{\partial x}\right)^T \eta \left(\frac{\partial x'}{\partial x}\right) = \Omega^2(x)\, \eta$$

Conformal group: $SO(d,2)$

Euclidean conformal group $(\eta_{\mu\nu} \to \delta_{\mu\nu})$: $SO(d+1,1)$

$so(d,2) =$ Poincare

+ "special conformal" $K_\mu = 2x_\mu (x^\alpha \partial_\alpha) - x^2 \partial_\mu$

+ dilatation $\mathcal{D} = x^\mu \partial_\mu$

/scale transf.

AdS/CFT

quantum gravity in AdS_{d+1} $=$ CFT_d

"bulk" "boundary"

ie,

$$\mathcal{H}_{grav} = \mathcal{H}_{CFT}$$

$$\mathcal{O}_{grav} \longleftrightarrow \mathcal{O}_{CFT}$$

Thermo:

$$Z_{cft}(\beta) = \mathrm{Tr}_{\mathcal{H}_{CFT}} e^{-\beta H}$$

$$= Z_{grav}(\beta)$$

$$\approx \sum_{\text{saddles } \bar{g}} e^{-I_E(\bar{g})} Z_{QFT}(\bar{g})$$

more generally,

Z_{grav} w/ boundary condition

$$ds^2 \to \frac{\ell^2}{z^2}\left(dz^2 + h_{ij}(\vec{x})\, dx^i dx^j\right) + \mathcal{O}(z^{d-2})$$

$$\phi \to z^{\#} J(\vec{x}) + \text{subleading}$$

$$= Z_{cft}[h_{ij}, J]$$

fixed metric CFT lives on

other background fields

$$I_{cft} \to I_{cft} + \int d^d x\, \mathcal{O}(x) J(x)$$

"GKPW dictionary"

Comments

* large N_{dof} in CFT

to account for $S_{BH} = \frac{\text{Area}}{4}$

$$N^2 \sim \left(\frac{\ell_{AdS}}{\ell_{planck}}\right)^{d-1}$$

But sparse: $\mathcal{O}(1)$ states @ low energies

* strong / weak

Example: Transport; entropy density

* Conformal symmetries of CFT on $\mathbb{R}^{d-1,1}$

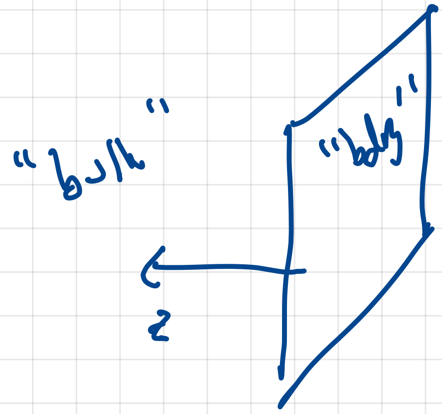
$\equiv$ isometries of AdS_{d+1}

Ex. AdS metric $ds^2_{bulk} = \frac{l^2}{z^2}\left(dz^2 - dt^2 + d\vec{x}^2\right)$

$z \to \lambda z$

$t \to \lambda t$

$\vec{x} \to \lambda \vec{x}$



ds^2_{bulk} invariant.

Consider a hypersurface @

$z = \varepsilon$

Induced metric

$$ds^2_{bdry} = \frac{l^2}{\varepsilon^2}\left(-dt^2 + d\vec{x}^2\right)$$

$$ds^2_{bdry} \to \underbrace{\lambda^2 \frac{l^2}{\varepsilon^2}}_{\Omega^2(x)}\left(-dt^2 + d\vec{x}^2\right)$$

* Scale transf. in CFT

$\equiv$ moving in radial direction in AdS (z)

* "deep in bulk" $\sim$ IR in CFT

"near bdry" $\sim$ UV in CFT

[roughly!!!]

The more careful statement is that taking a fixed object + moving in z moves it into UV/IR by scale transformation.

So for example high-E scattering deep into bulk can probe UV of CFT!

* "Top-down"

ex IIB strings on $AdS_5 \times S^5$

$= \mathcal{N}=4$ Super Yang-Mills

etc.

"Bottom up"

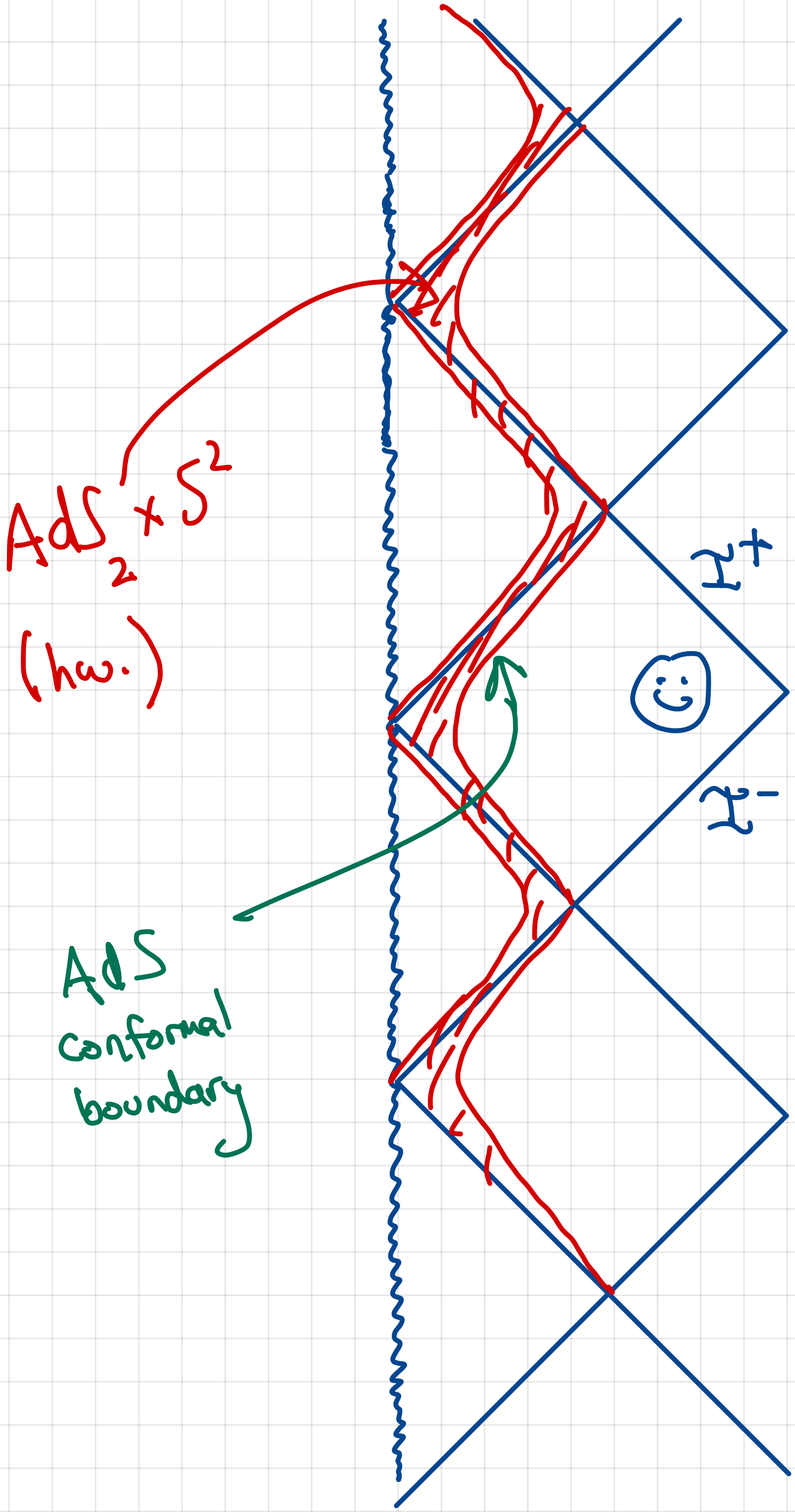
Semiclassical EFT in AdS $\leftrightarrow$ sector of large-N CFT

ex. AdS/CMT - microscopic theory of high-T_c cuprates is NOT IIB strings.

Near-Horizon / Decoupling limit

RN black hole in asymp.-flat,

with $Q = M$:



Similarly, "black branes" have near-horizon

$$AdS_{d+1} \times M$$

"outside view"

An observer ☺ cannot distinguish between