Statistics and wormholes in the conformal bootstrap

Lectures at Bootstrap 2022, Porto.

Tom Hartman, July 2022.

Selected Refs

ETH review   D'Alessio et. al.   1509.06411

ETH in CFT   Dymarsky, Lashkari, Liu   1610.00302

Universal $|C_0|^2$ Collier, Maloney, Maxfield, Tsiares   1912.00222

Wormholes vs. ETH

Saad Shenker Stanford   1903.11115

Saad   1910.10311

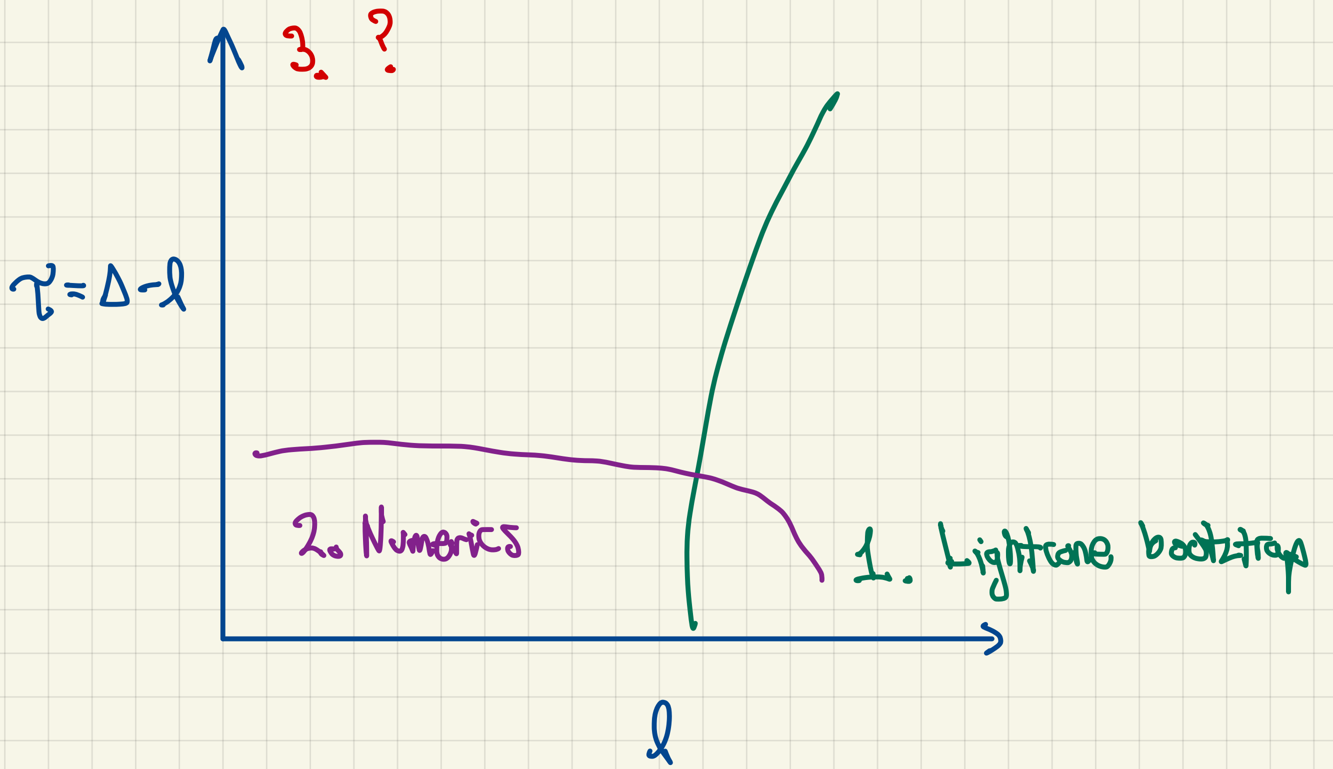
3d gravity as large-c ensemble

Chandra, Collier, Hartman, Maloney   2203.06511

S.

Intro

The spectrum of a CFT is organized by spin and twist:



What can we say about regime #3?

Anything??

Yes!

3. Thermo/hydro
Tauberian methods
RMT, ETH

These lectures will be about how we use these techniques to study CFT @ high energy.

We will focus on chaotic observables, unprotected by supersymmetry and not (heavily) constrained by integrability.

chaos

In this regime, often we are not trying to calculate individual Δ, ℓ, c_{ijk}, but statistics.

statistics

of spectrum:

$$\overline{\rho_\ell(\Delta)\rho_{\ell'}(\Delta')}\ , \qquad \overline{\rho\rho\rho}\ , \quad \text{etc.}$$

(energy level spacing statistics)

of matrix elements:

$$\overline{c_{ijk}\, c_{\ell mn}}\ , \qquad \overline{ccc}\ , \qquad \text{etc.}$$

of more complicated observables:

$$\overline{G(z,\bar{z})^2}\ , \ \ldots$$

where "$\overline{}$" = average over

- nearby states (microcanonical $\rho(E)$; Cardy; Tauberian)
- CFTs (conf. manifold, discrete family)
- "CFT data" Δ_i, c_{ijk}

ie random spectrum and random matrix elements

These $\wedge$ are both aspects of quantum chaos.

In these lectures we will focus mostly on matrix elements, for reasons I'll explain later.

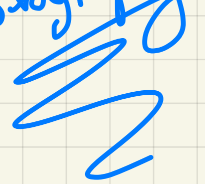
A big part of this problem is to understand how to phrase the question →

What's universal?

What's tractable?

For that we'll rely heavily on holography; recent progress in uncovering statistical aspects of black holes.

holography



A good long-term goal for the bootstrap is to bridge the gap between lightcone, numerics, hydro, RMT.

cf. Dodelson-Zhiboedov

But this is a subject that is just getting off the ground, lots to be done!

Message At large T, we mostly care about statistics.

These statistics can be probed by bootstrap and in AdS/CFT relate to higher topology spacetimes.

Plan

ETH

C_{ijk} in 2d CFT (combine ETH + Bootstrap)

Large-c ensemble

Wormholes

$\vdots$

Eigenstate Thermalization Hypothesis

ETH asserts that in a chaotic quantum system,

In chaotic regime, (in QM)

$$\langle m|\mathcal{O}|n\rangle = \langle \mathcal{O}\rangle_{E_m}\,\delta_{mn} + F(E_m,E_n)\,R_{mn}$$

energy basis — microcanonical — smooth — "random" and approx Gaussian

$$\overline{R_{ij}R_{kl}} \sim \delta_{ik}\delta_{jl} + \text{perms} + e^{-\#S}$$

Here I assumed only energy is conserved - e.g. spin chain QM.

When does it apply? That's very difficult to answer.

Best $\boxed{\text{evidence is numerical.}}$

Heuristic:

- theory is chaotic
- H non-degenerate
- $|m\rangle, |n\rangle$ have high energy ~ hydro, local therm.
- $\mathcal{O}$ is "simple" enough

(obviously H itself doesn't work)

Self-consistency of ETH requires

$$\langle m|\mathcal{O}\mathcal{O}|m\rangle \sim \langle \mathcal{O}^2\rangle_{E_m}$$

$\sum |n\rangle\langle n|$

$$\sum_n |\mathcal{O}_{mn}|^2 = \mathcal{O}(1)$$

This is a sum over e^S terms. So

$$\Rightarrow \mathcal{O}_{mn} \sim e^{-S/2}$$

$\therefore$ wlog

$$F(E_m, E_n) = e^{-S(\bar{E})/2} f(\bar{E}, \omega)$$

$$\bar{E} = \frac{1}{2}(E_m + E_n)$$

$$\omega = E_m - E_n$$

The fact that we used $\bar{E}$ is just a choice.

(absorb into f)

Note $f(E,\omega)$ is typically strongly peaked @ $\omega \sim 0$.

First term $\rightarrow$ Equilibrium

2nd term $\rightarrow$ Linear response

Thermalization

$$\frac{1}{2T}\int_{-T}^{T} dt \ \langle \psi | \mathcal{O}(t) | \psi \rangle = \sum_{m,n} \psi_m^* \psi_n \, \mathcal{O}_{mn} \ \underbrace{\frac{1}{2T}\int dt \, e^{it(E_{mn})}}_{\to \delta_{mn}}$$

$$\to \sum_n |\psi_n|^2 \, \mathcal{O}_{nn}$$

We expect an isolated, chaotic quantum system in a pure state to act "thermal" @ late times.

ETH $\Rightarrow$ $\sum_E \rho_\psi(E) \, \mathcal{O}(E)$ approaches equilibrium @ late time.

ETH "explains" this behavior: <u>each</u> term in the sum is thermal, individually.

An alternative is to say you can only prepare states with "random" $|\psi_n|^2$. This is just not how QM works. The best argument I know for this is numerical experiment.

(cf. integrable)

Hydrodynamics

For $\mathcal{O}$ a local operator,

$$G_\beta(x) = \frac{1}{Z(\beta)} \operatorname{Tr} e^{-\beta H} \mathcal{O}(x)\mathcal{O}(0) \overset{\text{ETH}}{\approx} \langle m|\mathcal{O}(x)\mathcal{O}(0)|m\rangle$$

↑ state w/ $E_m = E(\beta)$

$$G_\beta(\omega) \approx \int dt\, e^{-i\omega t} \langle m|\mathcal{O}(x)\mathcal{O}(0)|m\rangle$$

insert $\sum |n\rangle\langle n|$

Exercise: Show this sets

$$f(E,\omega) = \sqrt{G_{\beta(E)}(\omega)}$$

$$= \sqrt{2(1-e^{-\beta\omega})^{-1} \operatorname{Im} G^{Ret.}(\omega)}$$

linear response

i.e. smooth part is fixed by hydro

You can easily derive this by calculating 2pf in an eigenstate and $\langle m|\mathcal{O}(t)\mathcal{O}(0)|m\rangle = \langle \mathcal{O}(t)\mathcal{O}(0)\rangle_{E_m}$

In CFT:

$\langle m|\mathcal{O}|n\rangle = c_{\mathcal{O}mn}$

thermodynamic limit $V \to \infty$ (OR $N \to \infty$)

@ fixed E/V

$\Rightarrow \Delta_{m,n} \to \infty$

or $\Delta_{m,n} \sim N^2$ in holo. CFT (large gap)

$\therefore$ ETH suggests that in CFT, Roughly:

$C_{LHH} \approx \langle \text{heavy}|\mathcal{O}_{\text{light}}|\text{heavy}\rangle \sim$ thermal 1 p.f.

$C_{LHH'} \approx (\text{hydro}) * (\text{random matrix})_{HH'}$

But CFT has lots more structure than just QM - momentum conservation, crossing.

Heuristically, ETH tells us that these OPE coefficients should be "as random as possible" subject to other constraints.

end lecture #1

Challenge:

* What exactly does ETH + RMT predict for CFT spectrum and C_{ijk} in $d>2$?

* Does it hold? (numerics, holography)

Many of these results <u>should</u> apply in some way in higher dimensions.

But I'll mostly set $d=2$.

ETH-like ansatz for 2d CFT

Goal:

$$C_{ijk} = \underbrace{f(\Delta_i, \Delta_j, \Delta_k, \ell_i, \ell_j, \ell_k)}_{\text{bootstrap } |c_{ijk}|^2} \underbrace{R_{ijk}}_{\text{random}}$$

We'll derive this ansatz by inverting the OPE.

warm-up $C^{\text{all}}_{LHH'}$ ignoring descendants:

$$\langle O(z,\bar{z})\, O(0)\rangle_{\tau,\bar{\tau}}$$

$$= \sum_{\substack{i,j \\ [\underline{\underline{\text{all}}}\ \text{states}]}} \underbrace{O \overset{j}{\underset{i}{-\!\!\bigcirc\!\!-}} O}_{e^{2\pi i z h_i + 2\pi i(\tau - z)h_j}} \left|C^{\text{all}}_{O ij}\right|^2 \quad * \text{bars}$$

$\text{Im}\,\tau \to 0$

torus $\to$ cylinder

$z = \log w$

$\Rightarrow$

$$\approx G_{cyl} = e^{i\pi(\bar{h}-h)} \left(2\tau \sin\frac{z}{2\tau}\right)^{-2h} \left(2\bar{\tau} \sin\frac{\bar{z}}{2\bar{\tau}}\right)^{-2\bar{h}}$$

Invert by Fourier $\Rightarrow$

$$\overline{e^{S}\left|C^{\text{all}}_{O ij}\right|^2}^{\,\text{states}} \approx G_{cyl}\big(\underset{\substack{\uparrow \\ h_i - h_j}}{\omega},\ \underset{\substack{\uparrow \\ \bar{h}_i - \bar{h}_j}}{\bar{\omega}}\big)$$

(branch cuts are tricky... best way is to calculate
$\text{Im}\, G_R \sim \rho \sim G_+ - G_-$)

By inverting approximation we only get smeared answer.

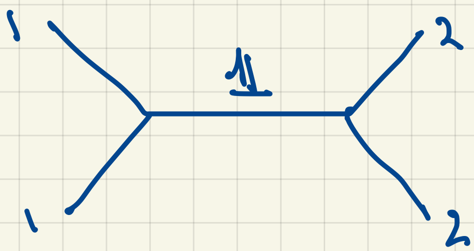
Now if we forgot Virasoro we'd recover ETH.

However this cannot be right, because obviously OPE coeff. of descendants are not independent.

To fix this, redo the calculation using the Virasoro crossing kernel.

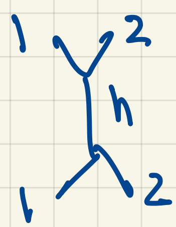
Now including Virasoro:

Define C_0 by:



Virasoro block $\mathcal{F}_{1122}(\mathrm{id}, 1-z)$

$$= \int dh \; \rho_0(h) \; C_0(h_1, h_2, h)$$



$\mathcal{F}_{1221}(h, z)$

Cardy

$$\rho_0(h) := \exp\left[2\pi\sqrt{\frac{c}{6}\left(h-\frac{c}{24}\right)}\right]$$

Ponsot-Teschner $\Rightarrow$

We'll return to the contour momentarily.

$$C_0(h_1,h_2,h_3) \sim \frac{C_{DOZZ}(h_1,h_2,h_3)}{\sqrt{\rho_0(h_1)\,\rho_0(h_2)\,\rho_0(h_3)}} \underbrace{\mathcal{N}}_{\mathcal{O}(c^0)}$$

Liouville OPE coeff.

(I'm dropping a 1-loop prefactor, see refs for exact.)

Why Liouville? Because Liouville is related to rep. theory of Virasoro. We are NOT discussing Liouville CFT!

<u>Contour:</u>

$$\text{“}\int dh\text{”} = \underset{\text{discretum}}{\text{multitwist}} + \int_{\frac{c-1}{24}}^{\infty} dh$$

"Virasoro MFT"

You can do this all exactly, but I'll just describe answer @ large c.

<u>Semiclassical limit</u> $c \to \infty$, $\frac{h}{c}$ fixed

$$h = \frac{c}{6}\eta(1-\eta)$$

$h < \frac{c}{24}$, $\eta \in [0, \frac{1}{2}]$ conical "defect"

$h > \frac{c}{24}$, $\eta = \frac{1}{2} + i\gamma$ "black hole"

<u>VMFT:</u>

If $\eta_1 + \eta_2 < \frac{1}{2}$:

$$\eta_{[\theta_1 \theta_2]} = \eta_1 + \eta_2$$

Otherwise:

no multitraces

Universal $|c_{ijk}|^2$

$$\langle \mathcal{O}_1(0)\,\mathcal{O}_2(z)\,\mathcal{O}_2(1)\,\mathcal{O}_1(\infty)\rangle \approx |\mathcal{F}(id, 1-z)|^2$$

when: $z \to 1$

<u>OR</u>

$c \to \infty, \quad z, \bar{z} > \frac{1}{2}$

invert
$\Rightarrow$

$$= \int dh \int d\bar{h} \;\; \rho_0(h)\rho_0(\bar{h}) \;\; C_0(h_1 h_2 h)\, C_0(\bar{h}_1 \bar{h}_2 \bar{h})\, |\mathcal{F}|^2$$

$\Rightarrow$

$$\boxed{\overline{|C_{ijk}|^2}^{\,\text{states}} \approx C_0(h_i, h_j, h_k)\, C_0(\bar{h}_i\, \bar{h}_j\, \bar{h}_k)}$$

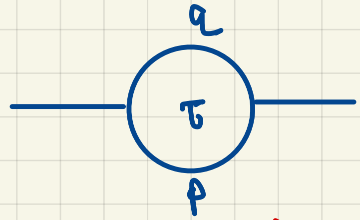
This derivation was for "$LL'H$"

i.e. $h_i \to \infty$

<u>or</u> $c \to \infty, \; h_i \gtrsim \frac{c}{12} \; \left(\frac{c}{24}\right)$

But <u>same</u> <u>formula</u> holds for LHH' and $HH'H''$

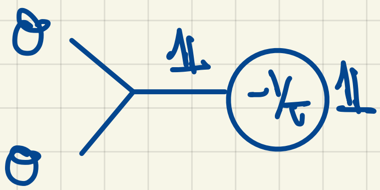
LHH'

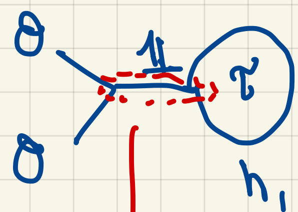
$$\langle \mathcal{O}(z,\bar z)\,\mathcal{O}(0)\rangle_{\tau,\bar\tau} = \sum_{p,q} \Big|$$  $$\Big|^2 \, |C_{\mathcal{O}pq}|^2$$

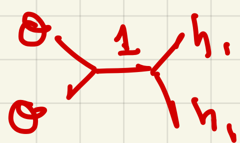
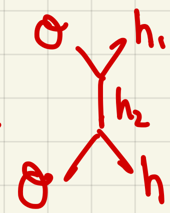
torus Vir block

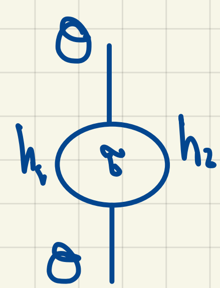
(cf. warm-up w/out descendants)

Im $\tau \to 0$ or $c \to \infty$

$$\approx \Big|$$  $$\Big|^2$$

$$= \Big| \int dh_1\, \rho_0(h_1)$$  $$\Big|^2$$

 $$= \int dh_2$$  $$\rho_0\, C_0$$

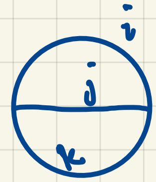
$$= \Big| \int dh_1 dh_2\, \rho_0(h_1)\rho_0(h_2)\, C_0(h_1,h_2,h_{\mathcal{O}})$$  $$\Big|^2$$

for LHH'

(ie agrees with boxed formula above)

Skipped

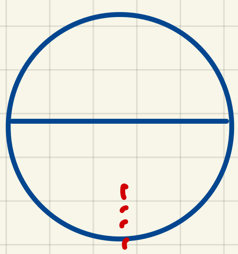
Homework: repeat for $HH'H''$

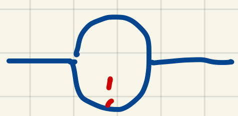
hint: $Z_{\text{genus } 2} = \sum$  $|c_{ijk}|^2$

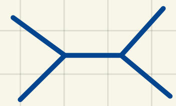
So the universal C_0 formula applies to all these cases.

It is useful to note these different regimes are related by analytic continuation:

Continue:

 $|C_{HH'H''}|^2$

$H'' \to$ light $\Rightarrow$  $|C_{LHH'}|^2$

$H' \to$ light $\Rightarrow$  $|C_{LLH'}|^2$

SKIP

Virasoro ETH

for $i \neq j \neq k$,

$$C_{ijk} = \sqrt{\underbrace{C_0(h_i\, h_j\, h_k)}_{\frac{C_{DOZZ}}{\sqrt{\rho_0\rho_0\rho_0}}}\; C_0(\bar{h}_i\, \bar{h}_j\, \bar{h}_k)}\;\; R_{ijk}$$

↑ replaces $e^{-S/2}\sqrt{G_{th}}$

$$\overline{R_{ijk} R_{lmn}} = \delta_{il}\,\delta_{jm}\,\delta_{kn} \pm \text{perms} + \text{subleading}$$

Gaussian:

$$\overline{RRRR} = \overline{RR}\;\overline{RR} + \cdots + e^{-\# S}$$

etc.

in "chaotic regime": any $h_i \to \infty$

or $c \to \infty$ w/ @ least one $h_i > \frac{c}{24}$ (black hole)

Summary

$$\text{ETH} \sim (\text{crossing kernel}) * (\text{random } \mathcal{O}(1))$$

So far: fixed CFT, particular R_{ijk}

Now: Average over R_{ijk}

Assume holographic CFT

(why? because of holography; motivated by SYK model)

(Actually $\overline{GG}$ below doesn't require holo CFT).

Call this

"Large-c Ensemble of CFT data"

end lecture #2

Large-c Ensemble vs. 3d gravity

Defects $\quad h \in \left(\frac{c}{32}, \frac{c}{24}\right)$

Black Holes $\quad h > \frac{c}{24}$

$\rho_0(h)\rho_0(\bar{h})$ Cardy

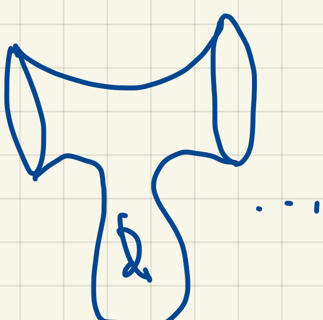
$$\overline{C_{ijk} C^*_{lmn}} = |C_0(h_i, h_j, h_k)|^2 \delta_{il}\delta_{jm}\delta_{kn} \quad \pm \text{ perms}$$

$$C_0 = \frac{C_{DOZZ}}{\sqrt{\rho_0\rho_0\rho_0}}$$

Motivation

2d JT gravity = RMT (SSS)

RMT: H is random



The type of avg. is different, but philosophy is similar.

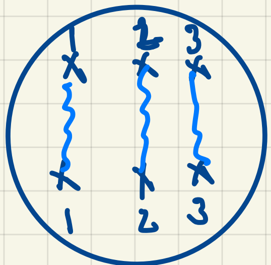
off-shell

Spectral data, level spacing requires off-shell QG; we will only require saddles for C_{ijk}.

Average Crossing Invariance of the ensemble

Ex. $G_6 = \langle O_1 O_2 O_3 O_3 O_2 O_1 \rangle$

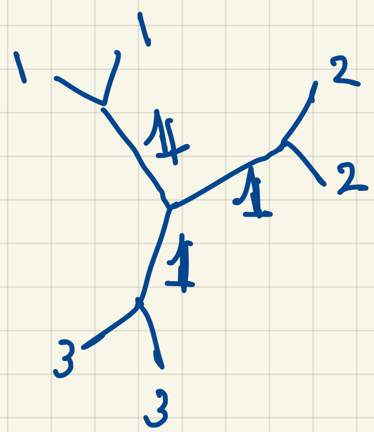
$G_6 \approx$



AdS_3

Conical Defects in AdS_3

$e^{-S_{grav}}$

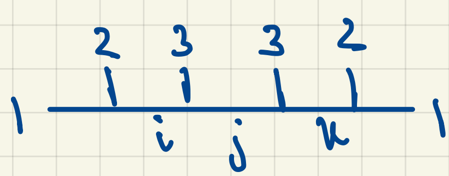
$\approx \Big|$ 

$\Big|^2$

Virasoro block

$|\mathcal{F}_{star}(\mathbb{1})|^2$

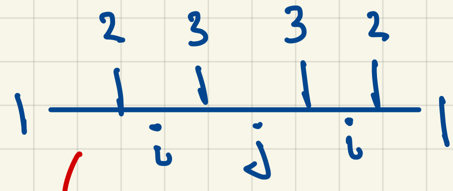
This should agree with other channels. For example, let's expand in comb channel:

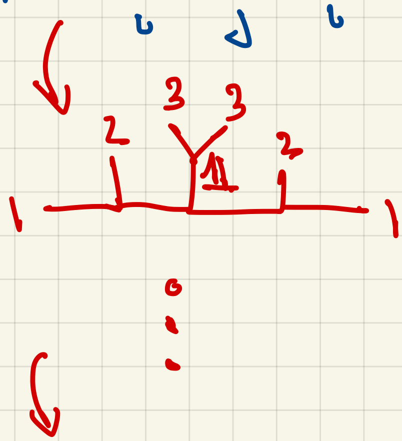
$$\overline{G_6} = \sum_{i,j,k} \Big|$$ 

$$\Big|^2 \overline{C_{12i} C_{3ij} C_{3j3} C_{3k2} C_{12k}}$$

$\delta_{ik} C_0^4$

$$\approx \iiiint \Big| \quad \Big|^2 \, |C_0|^4$$





$$\approx \left| \mathcal{F}_{star}(\mathbb{1}) \right|^2$$

<u>Point</u>: Gaussian avg. in heavy channel = identity in some other channel.

2-CFT observables $\overline{GG}$

$$G(z,\bar z) = \langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle$$

$$\overline{G(z,\bar z)\, G(z',\bar z')}_{\text{conn}}$$

← so we can ignore "diagonal"-type terms in ETH.

$$= \overline{\sum_i |C_{\mathcal{O}\mathcal{O}i}|^2 \left| \underset{z}{\succ_i \prec} \right|^2 \sum_j |C_{\mathcal{O}\mathcal{O}j}|^2 \left| \underset{z'}{\succ_j \prec} \right|^2}$$

δ_{ij}

and $\displaystyle\sum_i \to \int dh_i\, d\bar h_i\, \rho_0(h_i)\rho_0(\bar h_i)$

$$= \int dh_i\, d\bar h_i\, \rho_0(h_i)\rho_0(\bar h_i)\, |C_0(h,h,h_i)|^4\, |\mathcal{F}(h_i,z)\mathcal{F}(h_i,z')|^2$$

$$= \left| \int dh\, \rho_0(h_i)\, C_0(h,h,h_i)^2\, \mathcal{F}(h_i,z)\,\mathcal{F}(h_i,z') \right|^2$$

↑ $\dfrac{C_{DZZ}}{\sqrt{\rho_0(h_i)}}$

$$\boxed{\overline{G(z,\bar{z})\,G(z',\bar{z}')}_{\text{conn}} = G_{\text{Liouville}}(z,z')\,G_{\text{Liouville}}(\bar{z}',\bar{z})}$$

Note * swap!

* $\overline{\langle O_1 O_2 \cdots \rangle_g \langle O_1 O_2 \cdots \rangle_g} = G_L G_L'$

* does NOT require holo. CFT if $z \to 1$ (interpretation ??)

only assumed we are in a kinematic regime $(z,\bar{z})$ where ETH applies, and average over R_{ijk}.

So this is a true math fact about chaotic CFTs in the Tauberian limit; but I don't know what it's good for.

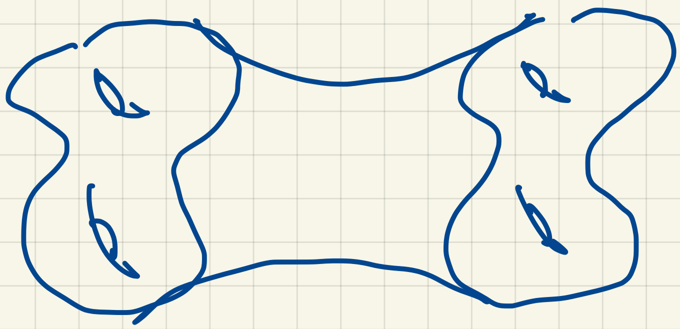
Challenge: what does this imply for individual, non-holographic CFT? I suspect something!

Wormholes in 3d gravity

There is of course a long history of matching bootstrap solutions to bulk physics.

However there is a puzzle: Einstein gravity + Matter has solutions with multiple, disconnected boundaries.

ex (MM)

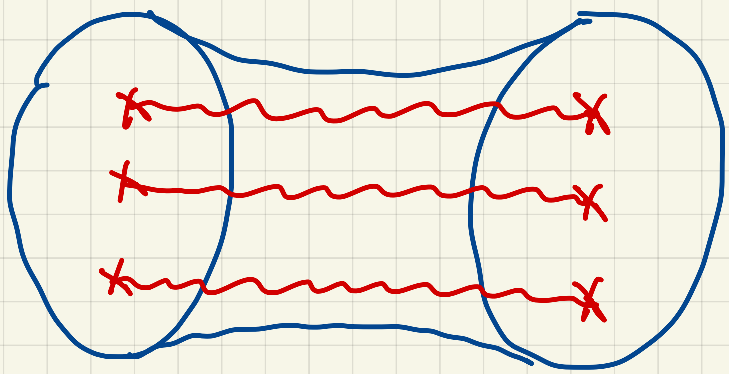


$$ds^2 = d\rho^2 + \cosh^2\rho \, \underbrace{d\Sigma^2}_{\text{any hyperbolic 2-mf.}}$$

$$\rightsquigarrow \quad Z_{g=2} Z_{g=2} \Big|_{conn} \quad ???$$

Claim: multiboundary solutions of 3d gravity + point particles = large-c ensemble.

ex.



Ansatz:

$$ds^2 = d\rho^2 + \cosh^2\rho \, e^{\Phi(z,\bar{z})} |dz|^2$$

(and defects sit @ fixed z_i)

Einstein eqns $\Rightarrow$

$$\partial\bar{\partial}\Phi = \frac{1}{2} e^{\Phi} - 4\pi G \sum_i \delta^{(2)}(z - z_i)$$

Liouville.

Now you can plug this ansatz into the Einstein action to calculate it on shell. The result is a Liouville action,

$$S_{\text{wormhole}} = V_\varepsilon - \frac{1}{2} A_\varepsilon + \begin{matrix}\text{defect}\\ \text{counterterms}\end{matrix}$$

$$\cong S_{\text{Liouville}}$$

$$e^{-S_{\text{wormhole}}} \approx \left| C_0(h_1, h_2, h_3) \right|^2 \qquad (z_i = 0, 1, \infty)$$

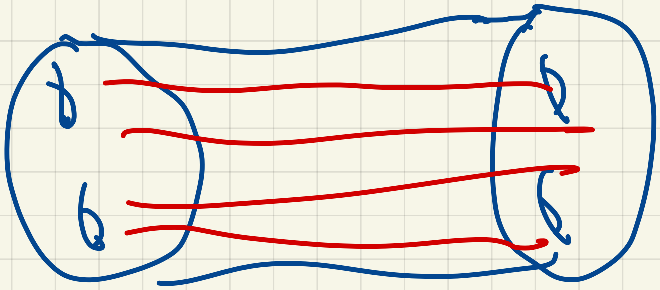
Note: Boundary metric is flat,

$$\rho \to \infty: \; ds^2 \sim d\rho^2 + e^{\Phi + 2\rho} \frac{1}{4} |dz|^2$$

$$\rho_c = \log\left(\frac{2}{\varepsilon}\right) - \frac{1}{2} \Phi$$

[except in little disks around defects]

General 2-boundary

$(z, \bar{z})$  $(z', \bar{z}')$

"Almost Fuchsian" z near z'

$$ds^2 = d\rho^2 + \cosh^2\rho \, e^{\Phi} \left| dz + \tfrac{1}{2}(1+\tanh\rho)\, \bar{t}(\bar{z})\, e^{-\Phi} d\bar{z} \right|^2$$

$\bar{t}(\bar{z})$ = quadratic differential on Σ

$$\text{e.g.} \quad \frac{1}{\bar{z}(\bar{z}-\bar{x})(\bar{z}-1)}$$

$$ds^2_{\text{left}} = |dz|^2$$

$$ds^2_{\text{right}} = |dz + \mu\, d\bar{z}|^2$$

$$e^{-S_{\text{wormhole}}} \approx \frac{G_{\text{Liou}}(z, z')\, G_{\text{Liou}}(\bar{z}', \bar{z})}{G_{\text{CFT}}(z, \bar{z})\, G_{\text{CFT}}(z', \bar{z}')}$$

Skipped

Derivation in 2 steps:

1) $z' = \bar{z}$: evaluate $S_{\text{wormhole}} \sim S_{\text{Liouville}}$

2) Insert extra $T(z)$
and show $\partial_{\bar{z}} T(z) = \partial_{\bar{z}'} T(z) = 0$

Branch Cuts

Skipped

The CFT quantity is Euclidean and single-valued.
The Liouville correlator is effectively Lorentzian

$G_{Liou}(z, z')$ is not single valued
as $z \circlearrowleft 0$ in Euclidean.

Bulk: Braiding

CFT: Corrections to C_0 from identity in other channels.
Euclidean braiding.

?

Skipped

Conjecture

Bulk Saddles $\xleftrightarrow{1\text{ to }1}$ Bootstrap channel + OPE contraction.

But to be clear, so far this is not systematic on either side of the relation.

Challenge: Prove it (topo recursion?)

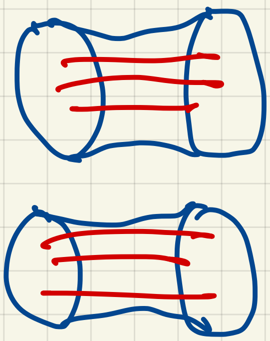
end lecture #3

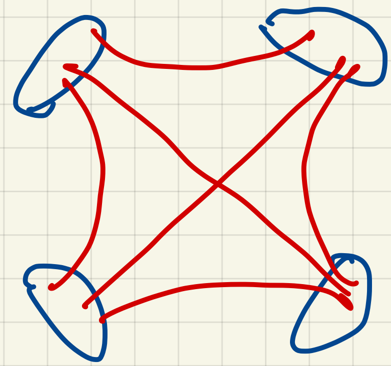
Many Boundaries ?

Skipped

$$\overline{CCCC}$$

$$= \overline{CC}\;\overline{CC} + \text{perms} + \text{non-gaussianities}$$





?

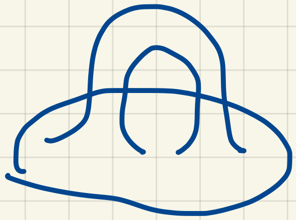
e^{-S} suppressed

Is this new input from gravity?

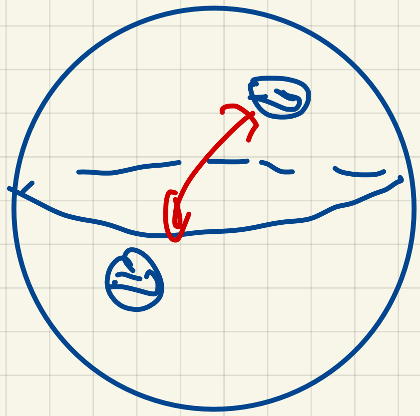
I doubt it. Probably follows from bootstrap, so can check bootstrap prediction from gravity.

Single-Boundary Wormholes ("Handles")

2d



3d



Coleman 1988:

wormholes $\rightarrow$ α parameters, baby universes

$$Z_{bulk} \sim \int D\phi \, \exp\left[-\int dx \, \mathcal{L} + \sum_i \int dx \, dy \, \mathcal{O}_i(x) \mathcal{O}_i(y)\right]$$

$$\sim \underbrace{\int d\alpha^i}_{\text{superselection sectors}} \underbrace{e^{-(\alpha^i)^2}}_{\text{Classical probability}} \int D\phi \, \exp\left[-\int dx \left(\mathcal{L} - \underbrace{\alpha^i}_{\text{Bulk coupling}} \mathcal{O}_i(x)\right)\right]$$

Coleman's argument is off-shell, ~~so~~ it's not under semiclassical control.

It is UV-sensitive.

This idea was mostly abandoned but revived recently in JT.

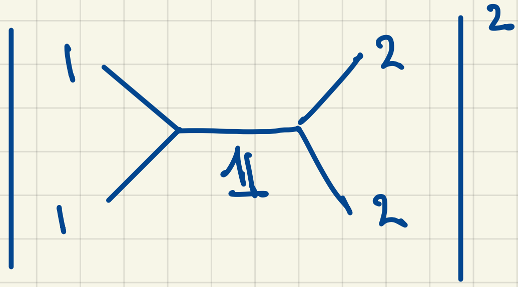
Now we'll show that the large-c ensemble has a similar effect.

In large-c ensemble,

Revisit

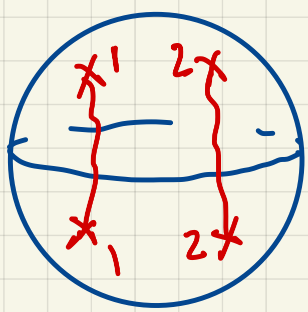
$$G = \langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_1 \rangle$$

$\approx$

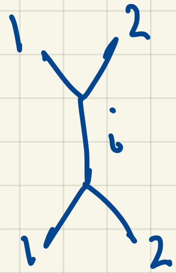


This term matches the bulk,

bulk



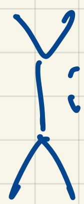
and it matches the Gaussian average in the dual channel.

$= \sum_{h_i > \frac{c}{24}}$ $\Big|$  $\Big|^2$ $\overline{|C_{12i}|^2}$

That sounds nice ...

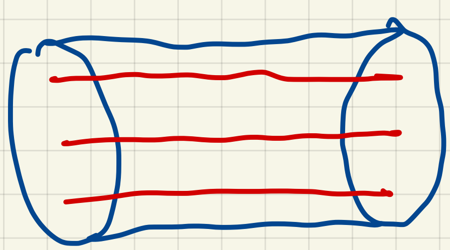
but we've neglected a term !

Missing :

$\sum_{h_i < \frac{c}{24}}$ $\Big|$  $\Big|^2$ $\overline{|C_{12i}|^2}$

defects

Nonzero because

 $= \overline{|C_{123}|^2}$

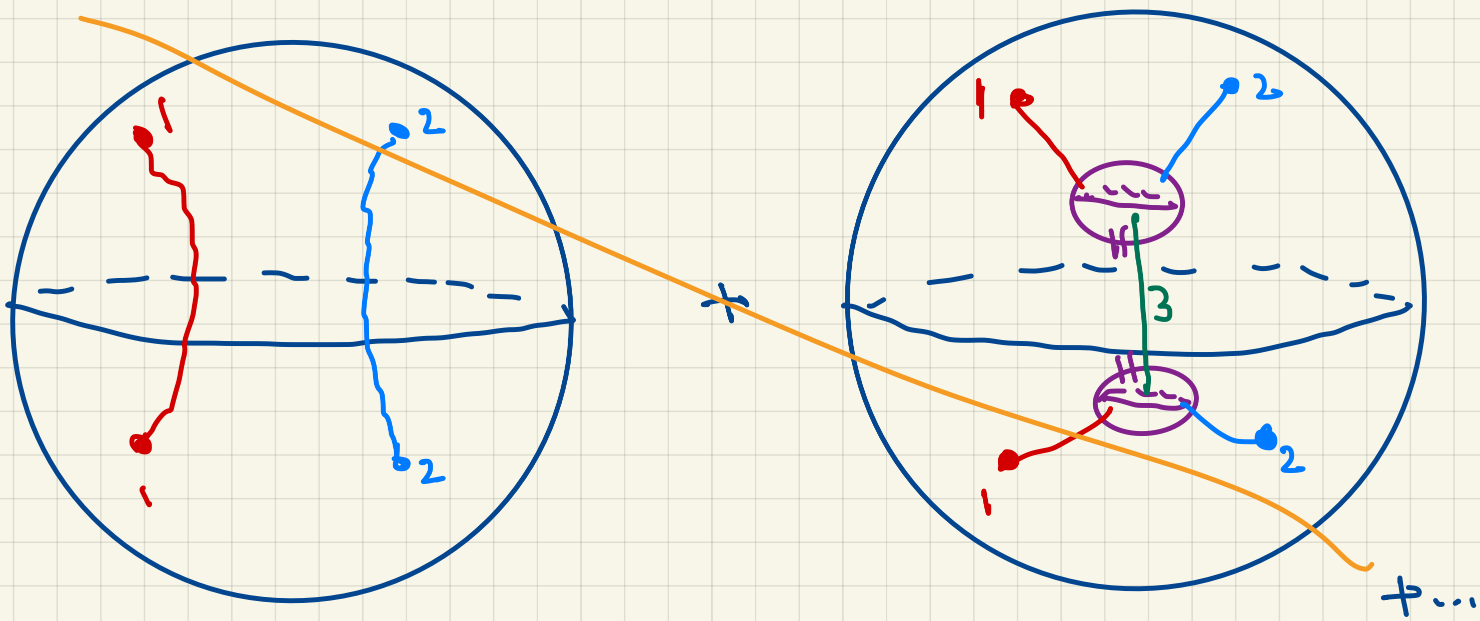
Thus for the whole story to hold together, we must find a <u>new</u>, <u>bulk</u> contribution to the 4_{pf}. with a single boundary!

Requires:

New contribution to bulk 4_{pf}. with exchange of defect i.

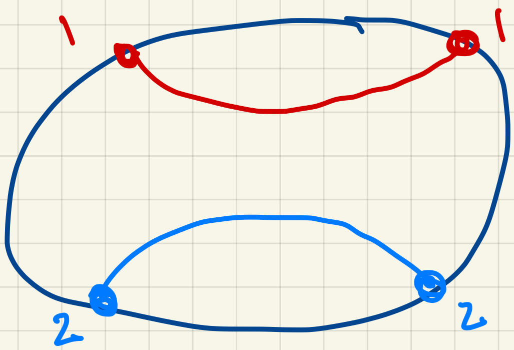
And indeed there is one. Here it is:

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_1 \rangle_{(\text{bulk})} =$$



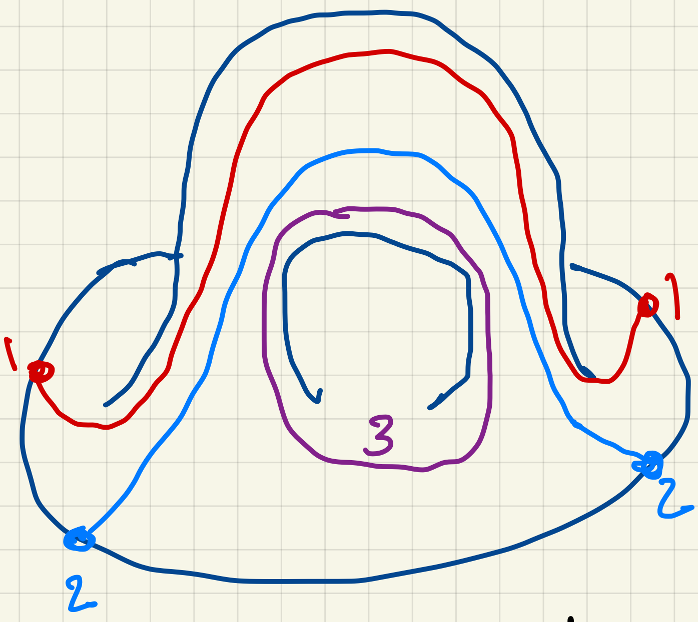
This is confusing, let's draw 2d cartoon

$$\langle O_1 O_2 O_2 O_1 \rangle_{(\text{bulk})} =$$



(Witten diagram + backreaction)

$+$

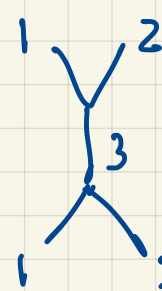


Same boundary cond!

Explain Quotient construction.

Calculate action:

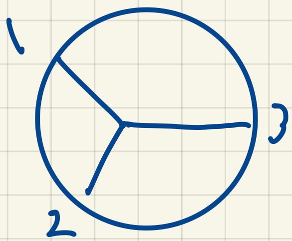
$$e^{-S} \approx |C_0(h_1,h_2,h_3)|^2 \left| \quad \right|^2$$



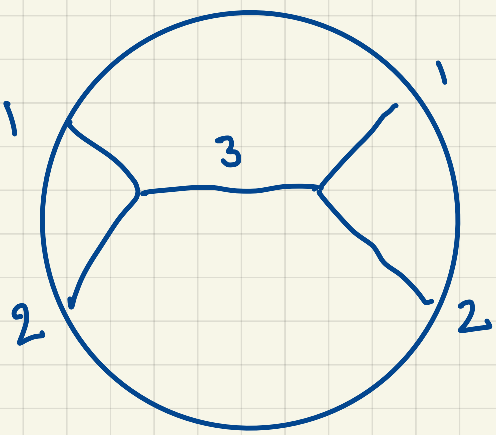
That's exactly what we needed from CFT!

Coleman-ology:

This is equivalent to adding a <u>bulk</u> coupling λ_{123} and averaging it:



$$\overline{\lambda_{123}} = 0$$



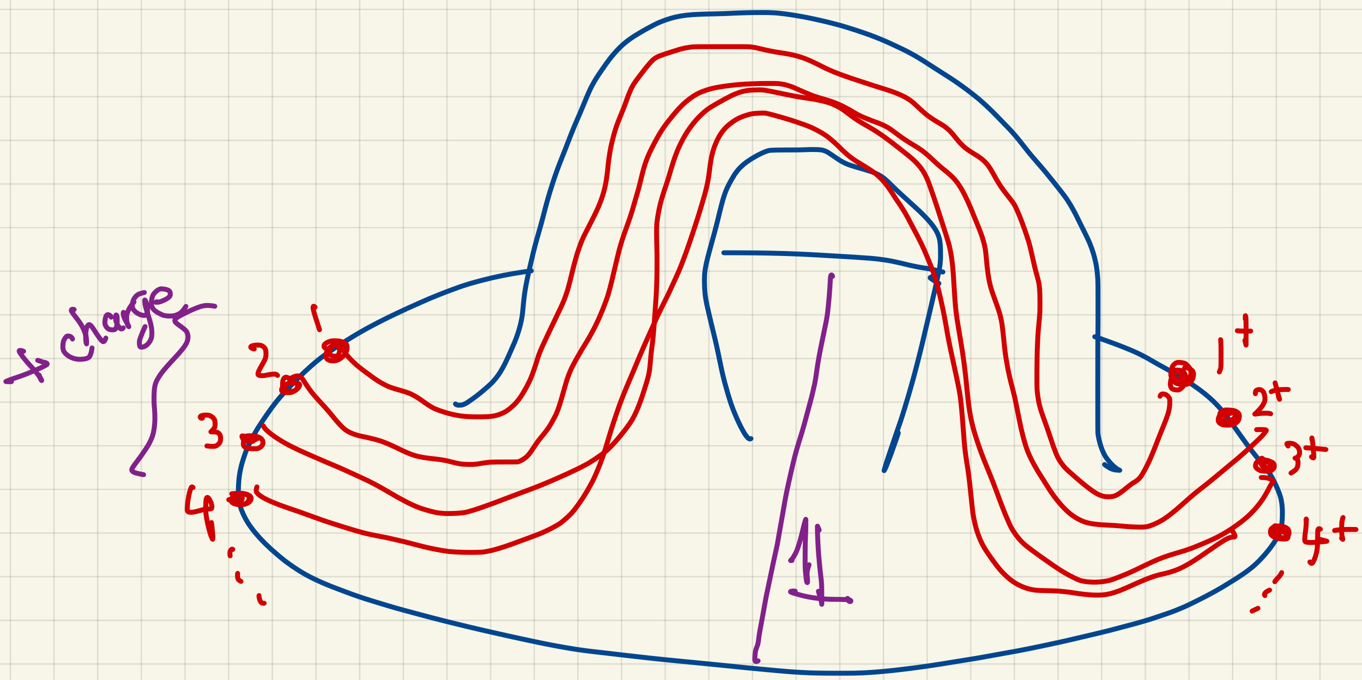
$$\overline{\lambda_{123}^2} = |C_0|^2$$

Does this mean 3d gravity has α-states?

Maybe so for pure gravity.

Probably not needed for UV-complete theories, where this is a low-E EFT calculation.

No global symmetries in QG



$G_8 \to |G_4|^2$ as Dist. $\to \infty$

wormhole $\neq 0$!

Because identity is running in this channel.

Non-factorization

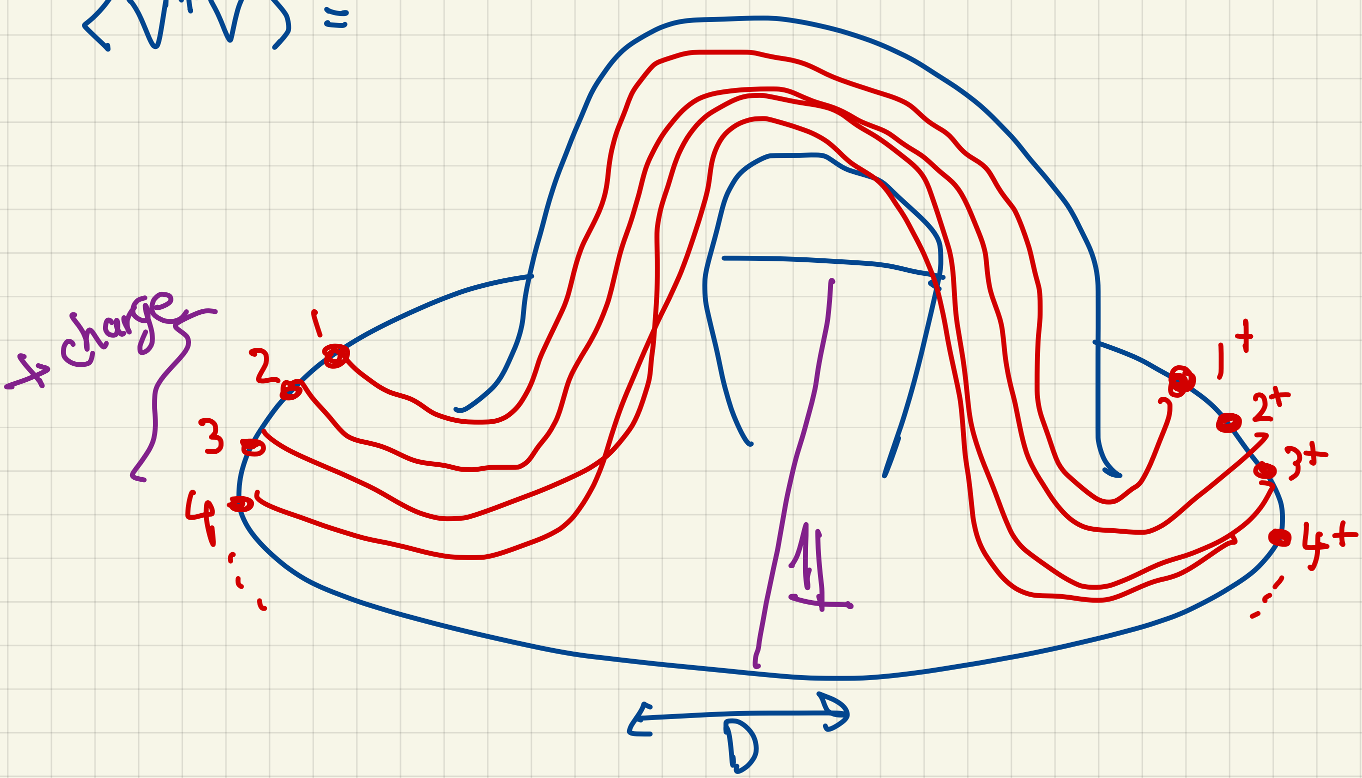
If V makes BH, expect

$$\langle V V' \rangle \sim e^{-S}$$

Let's confirm

$V = \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \ldots$

$\langle V^\dagger V \rangle =$



As $D \to \infty$, does not factorize $\to \langle V \rangle \langle V^\dagger \rangle$

suppose V is charged: $\langle V \rangle = 0$

but $\langle V^\dagger V \rangle \neq 0$!

no global symmetries in QG

But $\langle V' V \rangle = 0$

ensemble:

$\overline{\langle V' V \rangle} = 0$

$\overline{\langle V^\dagger V \rangle} = e^{-\# S}$

Interpretation (?): this calculates $\overline{\langle V' V \rangle}^{\text{fixed}} = 0$ unless $V' = V^\dagger$

in fixed CFT

This argument used the bulk, so it does not explain no-global-symmetries from CFT.

But it does give you a bootstrappy way to calculate the expected size of violations.

Outlook/Discussion

Why averaging?

* Universality / Low-E coarse graining

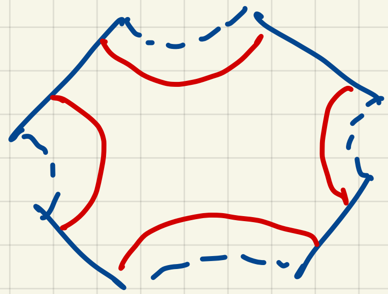
  ↪ But what is universal?

  (Hints from bulk / black holes!)

* Large-N limit (Witten-Schlenker)

Higher Dimensions ?

wormholes :



with spherical symmetry .

(So currently no insight on local operators.)

Lesson

Bootstrap / @ {multi-CFT observables}

We saw some examples, $\overrightarrow{GG}$

Obviously no more info in principle, but it is repackaged and perhaps more accessible to humans.

skip:

Math analogue:

$$\Delta(q) = q\prod_{n=1}^{\infty}(1-q^n)^{24}$$

modular cusp form of wt. 12

$$= \sum_{n=1}^{\infty}\tau(n)\,q^n$$

(Ramanujan τ)

Hardy 1927:

$$\overline{\tau(n)} \sim n^{11/2}$$

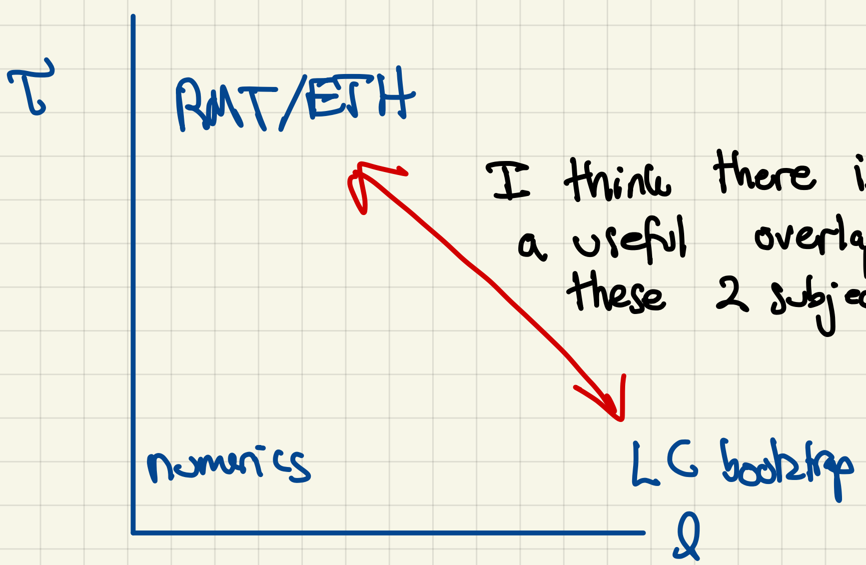
requires $|\Delta(q)|^2$

Questions

How much does low-energy gravity know about its UV completion?

Are <u>most</u> large-N CFTs holographic?

What does $\overline{GG}$ tell us in non-holo CFT?



I think there is very likely a useful overlap between these 2 subjects.

For example, to Mathematicians, "randomness" is very closely related to analyticity properties of partition function or its Mellin transform.